7 (1)

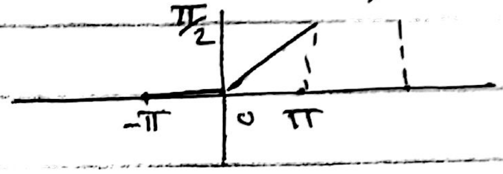
أ.د/ [illegible]

Model Answer Engineering Math. (2B)

Date 3/6/2015 Code: PME 1106 First year Elect.

Question no. (1) [illegible]

1) $F(x) = \begin{cases} 0 , & -\pi < x < 0 \\ \frac{x}{2} , & 0 < x < \pi \end{cases}$



$$a_0 = \frac{1}{L}\int_{-L}^{L} F(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} \frac{x}{2}\,dx\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} x\,dx = \frac{1}{2\pi}\,\frac{x^2}{2}\Big|_0^{\pi} = \frac{\pi}{4}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} F(x)\cos\frac{n\pi x}{L}\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_0^{\pi}\frac{x}{2}\cos nx\,dx\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} x\cos nx\,dx = \frac{1}{2\pi}\left[-\frac{x}{n}\cancel{\sin nx}^{\,0} + \frac{1}{n^2}\cos nx\right]_0^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{1}{n^2}(-1)^n - \frac{1}{n^2}\right] = \frac{(-1)^n - 1}{2\pi n^2}$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_0^{\pi}\frac{x}{2}\sin nx\,dx\right]$$

$$\begin{array}{ll} x & \oplus\ \sin nx \\ 1 & \ominus\ \frac{1}{n}\cos nx \\ 0 & \ \ -\frac{1}{n^2}\sin nx \end{array}$$

$$= \frac{1}{2\pi}\left[\frac{x}{n}\cos nx + \frac{1}{n^2}\sin nx\right]_0^{\pi}$$

$$= \frac{(-1)^n}{2n}$$

$$F(x) = \frac{\pi}{8} + \sum_{1}^{\infty}\left[\frac{(-1)^n - 1}{2\pi n^2}\cos nx + \frac{(-1)^n}{2n}\sin nx\right]$$

$x = 0$

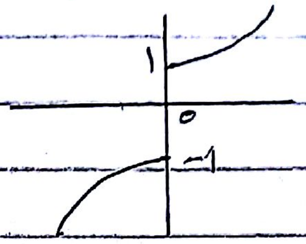
$$0 = \frac{\pi}{8} + \frac{1}{2\pi}(-2) - \frac{2}{18\pi} - - - -$$

$$\therefore \frac{\pi^2}{8} = 1 + \frac{1}{9} + \frac{1}{49} + - - -$$

(2)

2) $f(x) = e^{2x} \quad , \quad 0 < x < 1$

i) $a_0 = a_n = 0$



$$b_n = \frac{2}{L}\int_0^L f(x) \sin\frac{n\pi x}{L}\, dx$$

$$b_n = 2\int_0^1 e^{2x} \sin n\pi x\, dx$$

$$\int_0^1 e^{2x} \sin n\pi x\, dx = I$$

e^{2x} (+) $\sin n\pi x$

$2e^{2x}$ (−) $\frac{-1}{n\pi}\cos n\pi x$

$4e^{2x}$ (+) $\frac{-1}{n^2\pi^2}\sin n\pi x$

$$I = \frac{-1}{n\pi} e^{2x}\cos n\pi x + \frac{2}{n^2\pi^2} e^{2x}\sin n\pi x - \frac{4}{n^2\pi^2} I$$

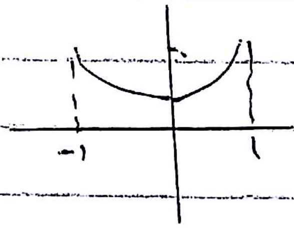
$$I = \frac{1}{1+\frac{4}{n^2\pi^2}}\left[\frac{-1}{n\pi} e^{2x}\cos n\pi x + \frac{2}{n^2\pi^2} e^{2x}\cancel{\sin n\pi x}\right]_0^1$$

$$I = \frac{n^2\pi^2}{n^2\pi^2+4}\left[\frac{-1}{n\pi} e^{2}(-1)^n - \frac{-1}{n\pi}\right]$$

$$I = \frac{n\pi}{n^2\pi^2+4}\left[-e^{2}(-1)^n + 1\right]$$

$$\therefore b_n = \frac{2n\pi}{n^2\pi^2+4}\left[-e^{2}(-1)^n + 1\right]$$

ii) $b_n = 0$



$$a_0 = \frac{2}{L}\int_0^L f(x)\, dx = 2\int_0^1 e^{2x}\, dx$$

$$\therefore a_0 = e^2 - 1$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\, dx$$

$$= 2\int_0^1 e^{2x}\cos n\pi x\, dx$$

e^{2x} (+) $\cos n\pi x$

$2e^{2x}$ (−) $\frac{1}{n\pi}\sin n\pi x$

$4e^{2x}$ (+) $\frac{-1}{n^2\pi^2}\cos n\pi x$

$$I = \int_0^1 e^{2x}\cos n\pi x\, dx = \frac{1}{n\pi} e^{2x}\sin n\pi x + \frac{2}{n^2\pi^2} e^{2x}\cos n\pi x - \frac{4}{n^2\pi^2} I$$

$$I = \frac{2}{\quad}\left[e^2(-1)^n - 1\right] \qquad \therefore a_n = \frac{4}{\quad}\left[e^2(-1)^n - 1\right]$$

Question No. (2)

1-i) $f(t) = \begin{cases} 0 & , 0 \le t < 4 \\ t^2 & , t > 4 \end{cases}$

$f(t) = 0\,[u(t-0) - u(t-4)] + t^2[u(t-4)]$

$f(t) = t^2 u(t-4) = [(t-4)+4]^2 u(t-4)$

$= (t-4)^2 u(t-4) + 8(t-4)u(t-4) + 16\,u(t-4)$

$\therefore F(s) = \frac{2!}{s^3} e^{-4s} + 8\frac{1!}{s^2} e^{-4s} + 16\frac{1}{s} e^{-4s}$

ii) $f(t) = \sin^2 t = \frac{1}{2} - \frac{1}{2}\cos 2t$

$F(s) = \frac{1}{2s} - \frac{1}{2}\frac{s}{s^2+4}$

2) i- $F(s) = \frac{2s-1}{s^3-s} = \frac{2s-1}{s(s-1)(s+1)}$

$\frac{2s-1}{s^3-s} = \frac{A}{s} + \frac{B}{(s-1)} + \frac{C}{(s+1)}$

$2s-1 = A(s-1)(s+1) + Bs(s+1) + C(s-1)s$

$s = 0 \qquad A = 1$

$s = 1 \qquad B = 1/2$

$s = -1 \qquad C = -3/2$

$F(s) = \frac{1}{s} + \frac{1}{2(s-1)} - \frac{3}{2(s+1)}$

$L^{-1} F(s) = f(t) = 1 + \frac{1}{2}e^{t} - \frac{3}{2}e^{-t}$

ii) $F(s) = e^{-3s}/s^3$

$F(s) = e^{-3s} \cdot \frac{1}{\ }$, $L^{-1}\frac{1}{\ } = t^2$

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(4)

3) $y' - 2y = 5e^{2t}, \quad y(0) = 1$

$$sY(s) - y(0) - 2Y(s) = 5\frac{1}{s-2}$$

$$(s-2)Y(s) - 1 = \frac{5}{(s-2)}$$

$$Y(s) = \frac{5}{(s-2)^2} + \frac{1}{(s-2)}$$

$$y(t) = 5e^{2t}t + e^{2t}$$

$$y(t) = (5t+1)e^{2t}$$

Question Number [3]

if $$\frac{\partial^2 E}{\partial t^2} + \alpha \frac{\partial E}{\partial t} + \beta E = \frac{\partial^2 E}{\partial x^2}$$

$$f(x+\lambda t) = e^{x+\lambda t}$$

$$\lambda^2 e^{x+\lambda t} + \alpha\lambda e^{x+\lambda t} + \beta e^{x+\lambda t}$$

$$= e^{x+\lambda t}$$

$$\lambda^2 + \alpha\lambda + \beta - 1 = 0$$

$$\lambda_{1,2} = \frac{-\alpha \pm \sqrt{\alpha^2 - 4(\beta-1)}}{2}$$

in The wave propagation

$$\alpha^2 - 4(\beta-1) < 0$$

$$4(\beta-1) > \alpha^2$$

$$\beta > \frac{\alpha^2}{4} + 1$$

① damped propagation

$$\alpha > 0, \quad \beta > \frac{\alpha^2}{4} + 1$$

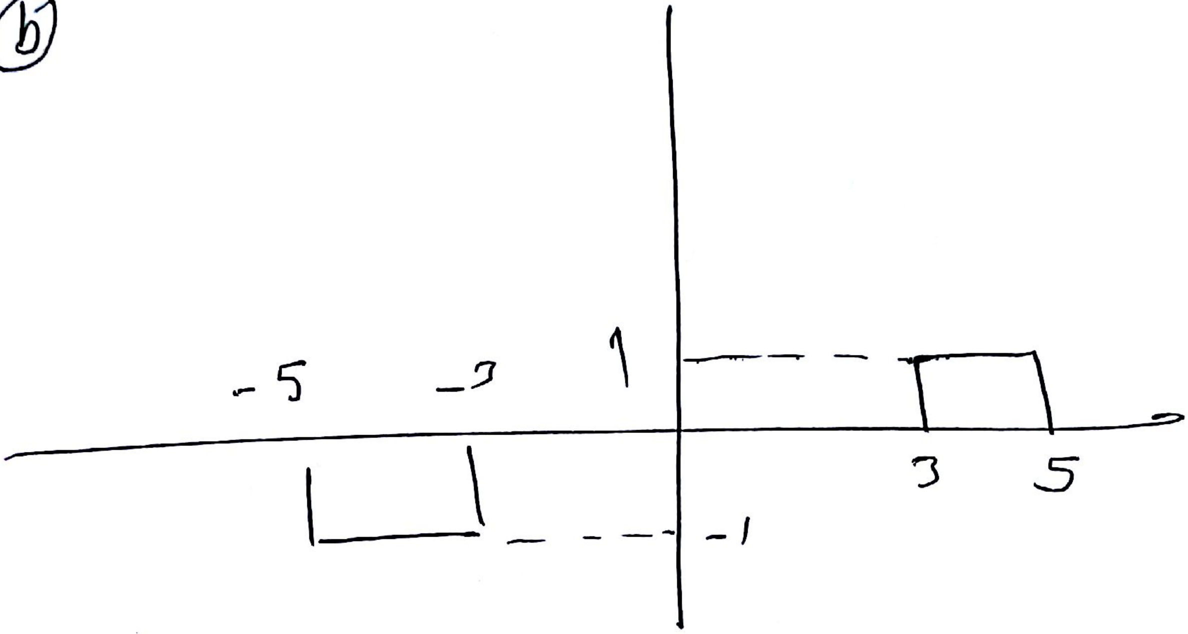
② grown propagation

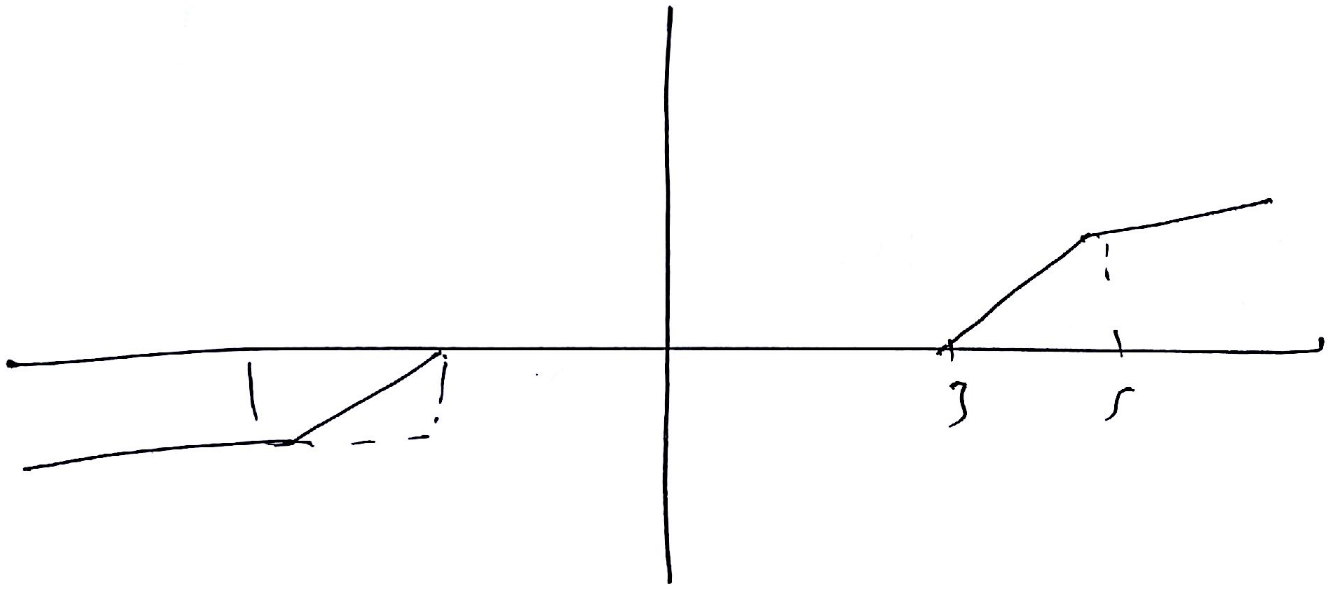
$$\alpha < 0, \quad \beta > \frac{\alpha^2}{4} + 1$$

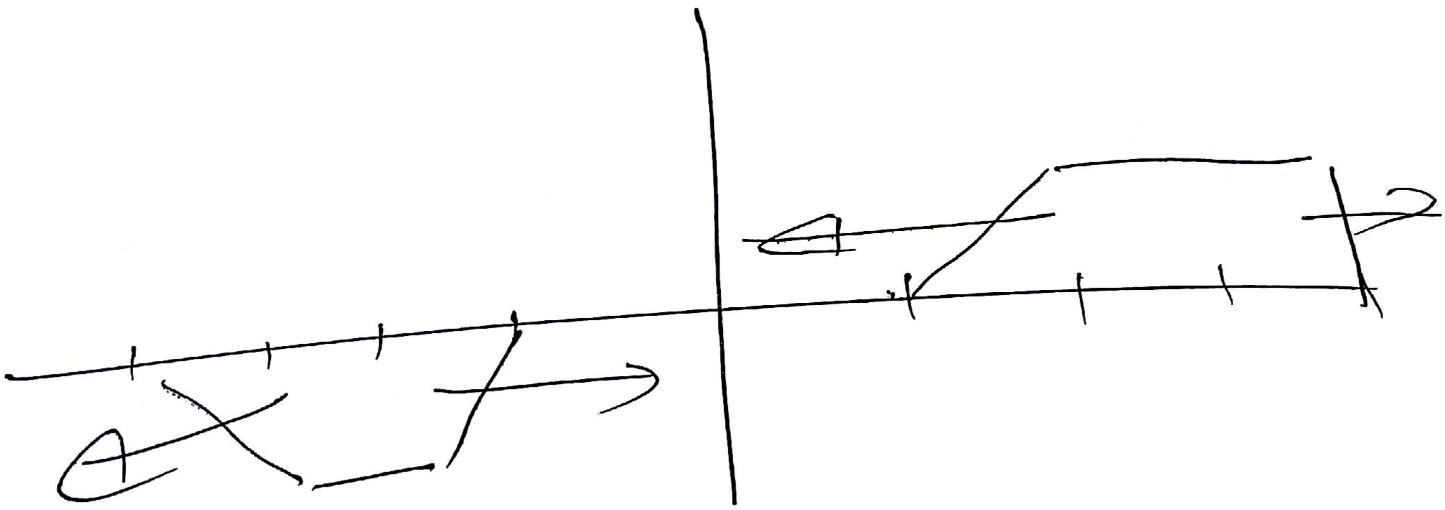
③ fixed propagation

$$\alpha = 0, \quad \beta > 1$$

(b)







2)

$$u_{xx} + u_{yy} = 0$$

$$u = f(x)\,g(y)$$

$$\frac{f''}{f} \cancel{=} -\frac{g''}{g} = \lambda$$

$$\frac{f''}{f} = \lambda, \quad \frac{g''}{g} = -\lambda$$

$$\lambda = 0$$

$$f = Ax + B, \quad g = Cy + D$$

refused

Question ④

ⓐ $x' = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} X$

$$\begin{vmatrix} 0-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} = 0$$

$$-\lambda(2-\lambda) + 1 = 0$$

$$\lambda^2 - 2\lambda + 1 = 0$$

$\lambda = +1, +1$ (ugtosc)

⑤

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$$

$$x = e^{\lambda t}$$

$$\lambda^2 + 6\lambda + 9 = 0$$

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Question 4

(a) $x' = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} x$

$$\begin{vmatrix} 0-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} = 0$$

$$-\lambda(2-\lambda)+1=0$$

$$\lambda^2-2\lambda+1=0$$

$$\lambda = +1, +1 \quad \text{(unstable)}$$

(b)

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$$

$$x = e^{\lambda t}$$

$$\lambda^2 + 6\lambda + 9 = 0$$

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$\lambda < 0 \qquad \lambda = -\omega^2$

$f'' + \omega^2 f = 0$

$f = A\cos\omega x + B\sin\omega x$

$g = C e^{\omega y} + D e^{-\omega y}$

$\lambda > 0, \qquad \lambda = \omega^2$

$f = A e^{\omega x} + D e^{-\omega x}$

$g = C\cos\omega y + D\sin\omega y.$

$$\lambda = -7, -3 \quad (\text{Stable})$$

2

$$\left| \begin{pmatrix} -(R_1 + R_2)/L_2 & R_2/L_2 \\ R_2/L_1 & -R_2/L_1 \end{pmatrix} - \lambda I \right| = 0$$

$$\begin{vmatrix} -11-\lambda & 3 \\ 3 & -3-\lambda \end{vmatrix} = 0$$

$$(11+\lambda)(3+\lambda) - 9 = 0$$

$$33+14\lambda+\lambda^2-9=0$$

$$\lambda^2+14\lambda+24=0$$

$$(\lambda+2)(\lambda+12)=0$$

$$\lambda=-2\ , \quad \lambda=-12.$$

Stable but it has not a periodic solution